

A Synergistic Approach for Abstracting Hardware Heterogeneity and Reducing Algorithmic Complexity: Powering HiCMA with oneAPI for HPC Applications

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Intel oneAPI DevSummit at SC21 (Live Virtual)
November 14, 2021



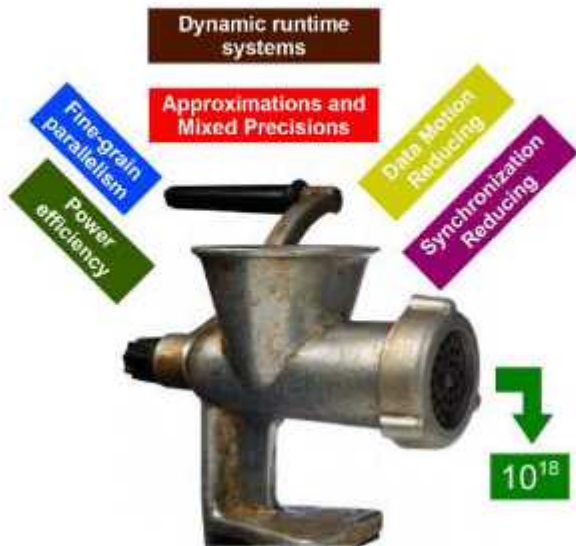
Acknowledgements

- ECRC: *S. Abdulah, K. Akbudak, R. Alomairy, M. Al Farhan, M. Genton, Y. Hong, D. Keyes, M. Ravasi, and Y. Sun*
- Paris Observatory: *F. Ferreira, D. Gratadour, A. Sevin*
- Research School of Astronomy & Astrophysics, Australian National University: *J. Cranney*
- Intel: *H. Ibeid, P. Thierry, A. Al-Jeshi, A. Abduljabbar*
- KAUST Supercomputing Lab: *S. Feki, B. Hadri, and S. Kortas*

HPC Recipe For Exascale Computing



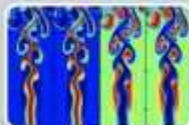
HPC Recipe For Exascale Computing



A Hostile Hardware Landscape

Architecture Specialization for Science

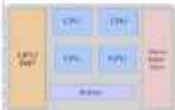
(hardware is design around the algorithms) can't design effective hardware without applied math



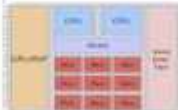
Past - Homogeneous Architectures



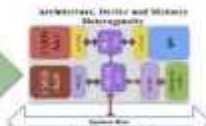
Present - CPU+GPU



Present - Heterogeneous Architectures



Future - Post CMOS Extreme Heterogeneity

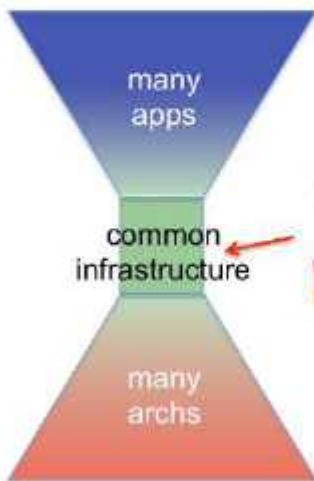


This needs to be done in close collaboration with applied mathematics

You cannot specialize effectively without deep understanding of the algorithmic target for those specializations
Need to know degrees of freedom for reformulating the mathematics to match hardware strengths

Slide courtesy from John Shalf, LBNL

The Hourglass Revisited



ECRC is
right
here



@KAUST_ECRC



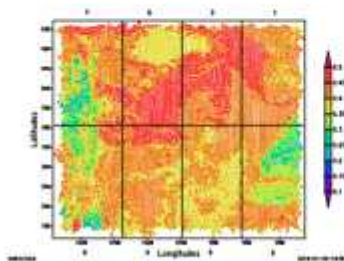
<https://www.facebook.com/ecrc@kaust>

Performing Climate/Weather Forecasting Simulations

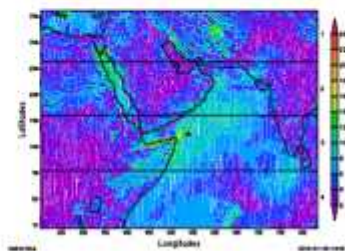
- Computational statistics: multivariate large spatial data sets in climate/weather modeling:

$$\ell(\boldsymbol{\theta}) = -\frac{1}{2}\mathbf{Z}^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta})\mathbf{Z} - \frac{1}{2}\log|\boldsymbol{\Sigma}(\boldsymbol{\theta})|$$

(a) Problem Definition:



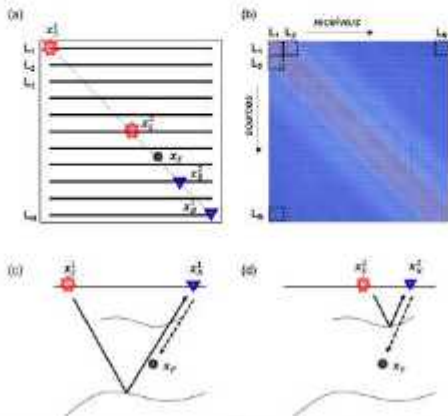
(b) Soil moisture.



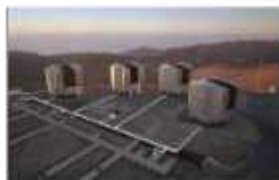
(c) Wind speed.

Revealing the Underground Layers with Seismic Imaging

- Transitioning to new energy technologies requires an even higher level of details to make informed decision
- Monitoring permafrost degradation
- Seismic redatuming is key!



Outsmarting the Atmospheric Turbulence in AO



(a) VLT.



(b) Subaru.



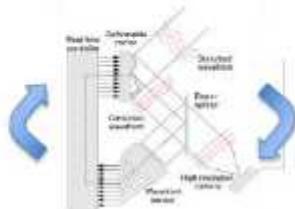
(c) E-ELT.



(d) ScExAO.

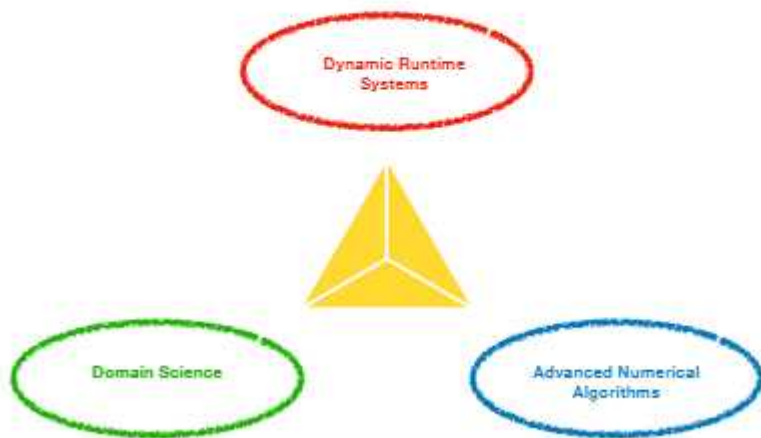


(e) MAVIS.



(f) Soft/Hard RTC.

An Effective Approach Based on a Separation of Concerns



Exploiting the Data Sparsity of these Matrices

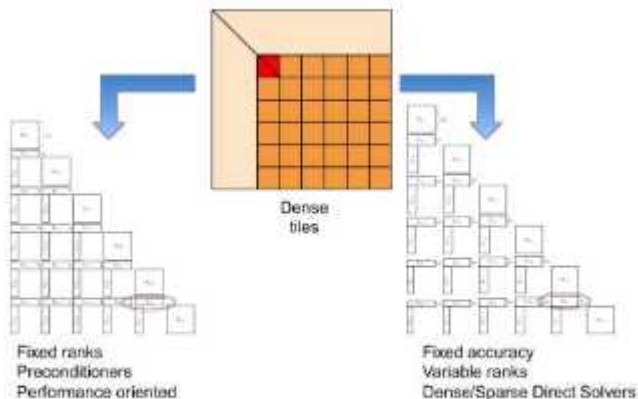
- Ubiquitous in computational science and engineering
- Symmetric, positive-definite matrix structure
- Apparently dense matrices but often data-sparse
- Decay of parameter correlations with distance

The HiCMA Library



Available at <http://github.com/ecrc/hicma>

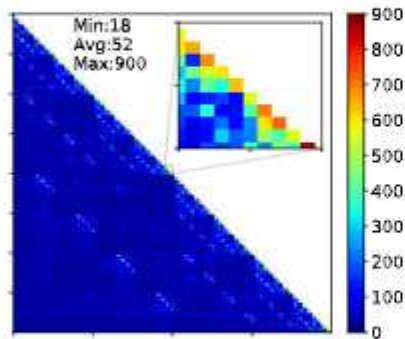
Tile Low-Rank Data Format as a Pragmatic Approach



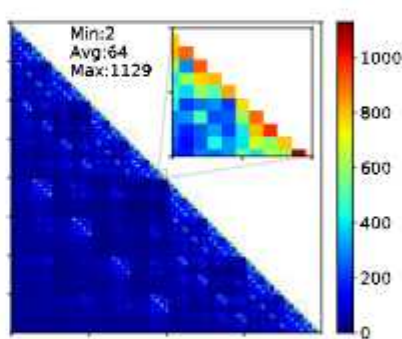
T. Mary, PhD Dissertation, Block Low-Rank multifrontal solvers: complexity, performance, and scalability, 2017.

C. Weisberger, PhD Dissertation, Improving multifrontal solvers by means of algebraic Block Low-Rank representations, 2013.

Rank Heatmaps of 3D Exponential Kernel with $N = 1M$, $b = 2700$, and $acc = 1e - 8$



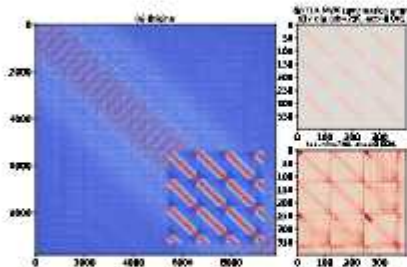
(a) Initial ranks.



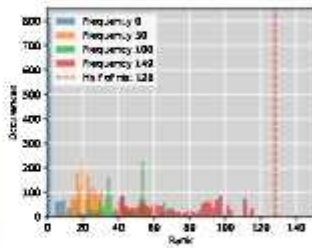
(b) Final ranks.

Q. Cao, Y. Pei, K. Akbudak, G. Bosilca, H. Ltaief, D. Keyes, and J. Dongarra: Leveraging PaRSEC Runtime Support to Tackle Challenging 3D Data-Sparse Matrix Problems, IPDPS'21

Rank Analysis for Seismic Frequency Matrices



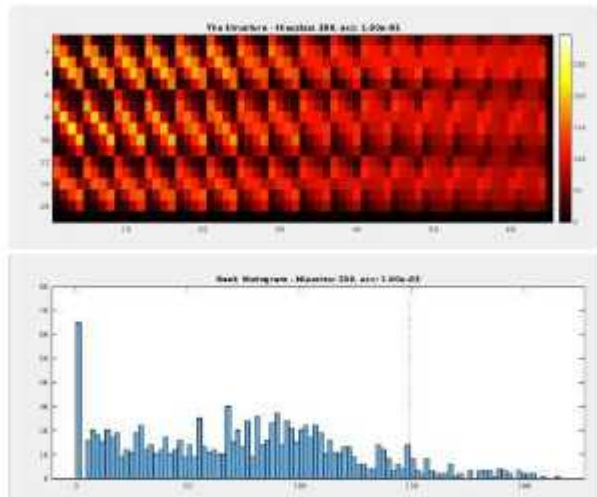
(a) Rank Heatmaps.



(b) Rank Distribution.

Y. Hong, H. Ltaief, M. Ravasi, L. Gattineau, D. Keyes, *Journal of Supercomputing Frontiers and Innovations*'21

Rank Analysis for MAVIS Instrument on the Very Large Telescope



Implementation Details: Programming Models

The logo for OpenMP, featuring the text "OpenMP" in a teal, sans-serif font. The word "Open" is in a regular weight, while "MP" is in a bold weight. A thick teal horizontal bar is positioned above the text, and a thin teal horizontal line is positioned below it. A small trademark symbol (TM) is located at the bottom right of the "P".

OpenMP™

The logo for oneAPI, featuring a stylized vertical stack of purple spheres connected by lines, resembling a molecular or atomic structure. A trademark symbol (TM) is located at the top right of the stack. Below the graphic, the text "oneAPI" is written in a bold, black, sans-serif font.

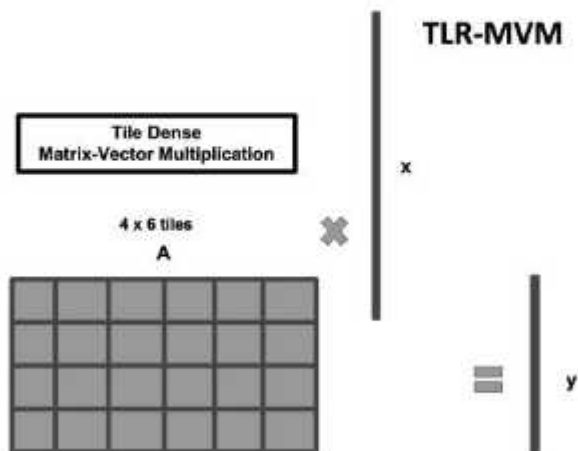
oneAPI™

Implementation Details: TLR Cholesky Factorization

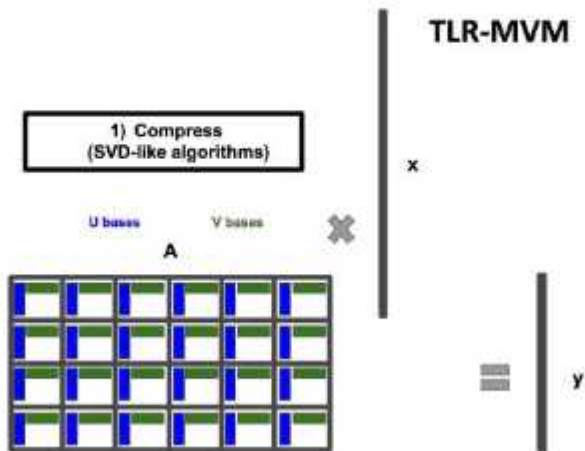
Algorithm 1 HiCMA_DPOTRF(HicmaLower, D, U, V, N, nb, rank, acc)

```
p = N / nb
for k = 1 to p do
  #pragma omp task depend(inout:D(k,k))
  hcore_dpotrf(HicmaLower, D(k,k), rank, acc)
  for i = k+1 to p do
    #pragma omp task depend(in:D(k,k)) depend(inout:U(i,k))
    hcore_dtrsm(V(i,k), D(k,k), rank, acc)
  end for
  for j = k+1 to p do
    #pragma omp task depend(in:U(j,k)) depend(in:V(j,k)) depend(inout:D(j,j))
    hcore_dsyrk(D(j,j), U(j,k), V(j,k), rank, acc)
    for i = j+1 to p do
      #pragma omp task
      depend(in:U(i,k)) depend(in:V(i,k))
      depend(in:U(j,k)) depend(in:V(j,k))
      depend(inout:U(i,j)) depend(inout:V(i,j))
      hcore_dgemm(U(i,k), V(i,k), U(j,k), V(j,k), U(i,j), V(i,j), rank, acc)
    end for
  end for
end for
```

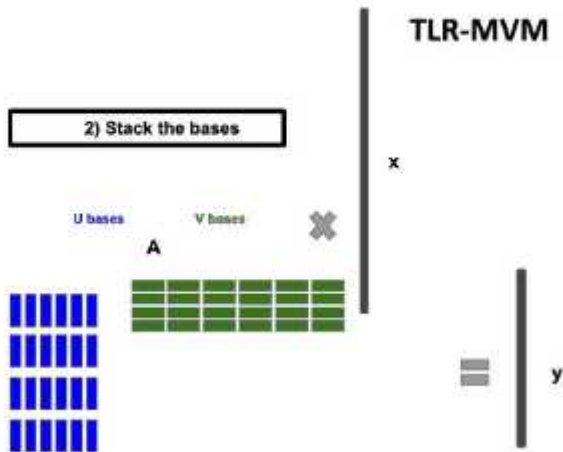
Implementation Details: TLR-MVM for MAVIS Instrument and Seismic Redatuming



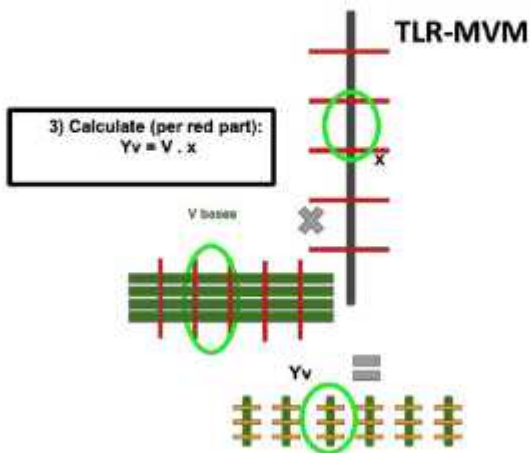
Implementation Details: TLR-MVM for MAVIS Instrument and Seismic Redatuming



Implementation Details: TLR-MVM for MAVIS Instrument and Seismic Redatuming



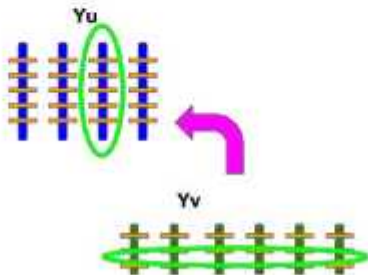
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Implementation Details: TLR-MVM for MAVIS Instrument and Seismic Redatuming

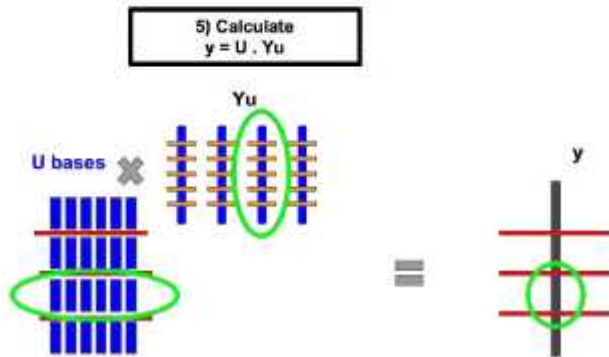
TLR-MVM

4) Translate
 Y_v (V bases) to Y_u (U bases)

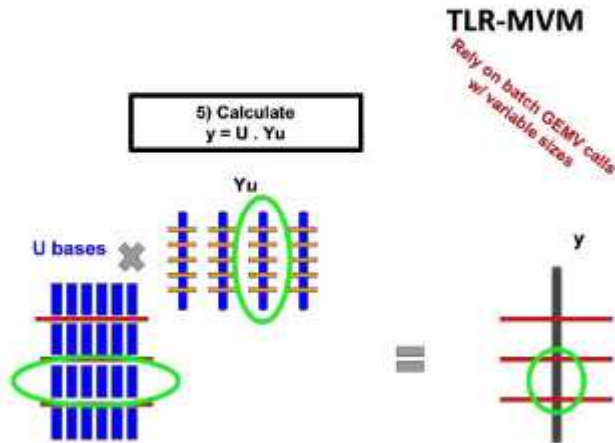


Implementation Details: TLR-MVM for MAVIS Instrument and Seismic Redatuning

TLR-MVM



Implementation Details: TLR-MVM for MAVIS Instrument and Seismic Redatuming



Implementation Details: TLR-MVM OpenMP for MAVIS Instrument and Seismic Redatuming

Algorithm 2 TLR-MVM_OpenMP(*Utiles*, *Vtiles*, *YuYvMapping*, *x*)

```
ntiles = N / nb, mtils = M / nb
#pragma omp parallel for
for k = 1 to ntils do
    cblas_gemv(Vtiles(k), x(k), yv(k)) // Phase 1: yv is output
end for
#pragma omp parallel for
for k = 1 to length of yv do
    yu(YuYvMapping(k)) = yv(k) // Phase 2: yu is output
end for
#pragma omp parallel for
for k = 1 to mtils do
    cblas_gemv(Utiles(k), yu(k), y(k)) // Phase 3: y is output
end for
```

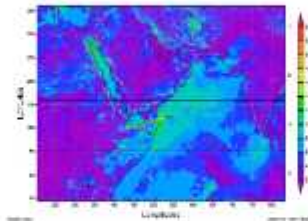
Implementation Details: TLR-MVM oneAPI for MAVIS Instrument and Seismic Redatuming

Algorithm 3 TLR-MVM_oneAPI(U_{tiles} , V_{tiles} , $Y_u Y_v \text{Mapping}$, x , devicequeue)

```
ntiles = N / nb, mntiles = M / nb
#pragma omp parallel for
for k = 1 to ntiles do
    oneAPI_gemv( $V_{\text{tiles}}(k)$ ,  $x(k)$ ,  $y_v(k)$ ,  $\text{devicequeue}$ ) // Phase 1:  $y_v$  is output
end for
devicequeue.wait()
devicequeue.parallel_for{ $y_u(Y_u Y_v \text{Mapping}(k)) = y_v(k)$ } // Phase 2:  $y_u$  is output
devicequeue.wait()
#pragma omp parallel for
for k = 1 to mntiles do
    oneAPI_gemv( $U_{\text{tiles}}(k)$ ,  $y_u(k)$ ,  $y(k)$ ,  $\text{devicequeue}$ ) // Phase 3:  $y$  is output
end for
devicequeue.wait()
```

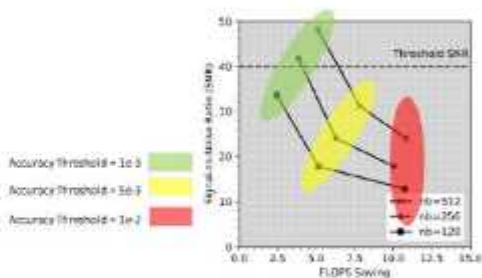
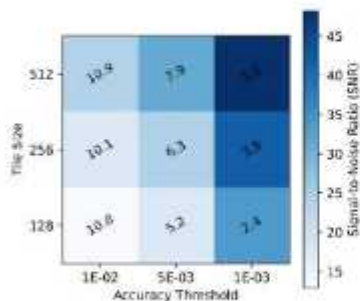
Numerical Accuracy Assessment: TLR Cholesky for MLE

- Estimation of the Matérn covariance parameters for four geographical regions of wind speed dataset.



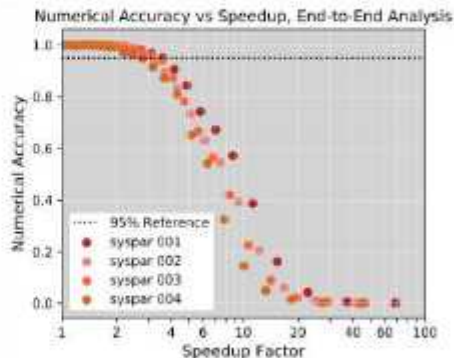
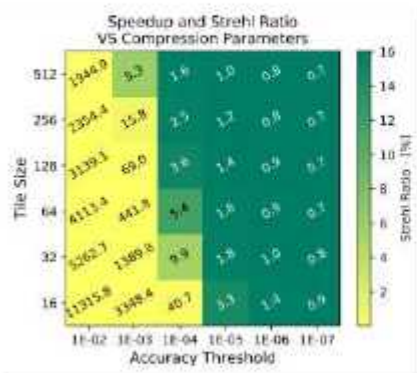
R	Variance (θ_1)				Matérn Covariance Spatial Range (θ_2)				Smoothness (θ_3)			
	TLR Accuracy			Full-ble	TLR Accuracy			Full-ble	TLR Accuracy			Full-ble
	10^{-5}	10^{-7}	10^{-9}		10^{-5}	10^{-7}	10^{-9}		10^{-5}	10^{-7}	10^{-9}	
R1	7.406	9.407	12.247	8.715	29.576	33.886	39.573	32.083	1.214	1.196	1.173	1.210
R2	11.970	13.159	13.550	12.517	26.011	28.063	28.707	27.237	1.290	1.267	1.260	1.274
R3	10.588	10.944	11.232	10.819	18.423	18.783	19.114	18.634	1.418	1.413	1.407	1.416
R4	12.408	17.112	12.368	12.270	17.264	17.112	17.247	17.112	1.168	1.170	1.168	1.170

Numerical Accuracy Assessment: TLR-MVM for Seismic Redatuming



Y. Hong, H. Ltaief, M. Ravasi, L. Gataineau, D. Keyes, *Journal of Supercomputing Frontiers and Innovations*'21

Numerical Accuracy Assessment: TLR-MVM for MAVIS Instrument



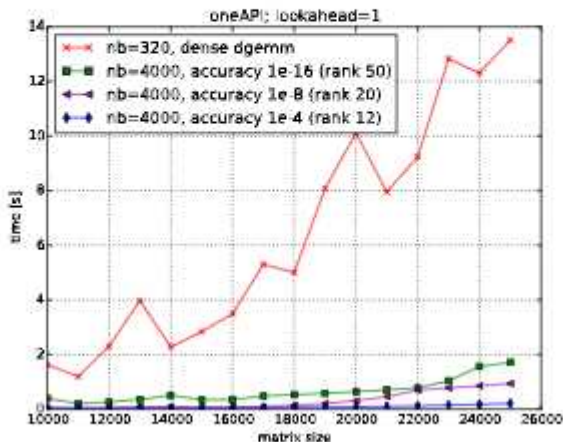
H. Ltaief, J. Cranney, D. Gratadour, Y. Hong, L. Gatineau, D. Keyes, SC'21

Environment Settings

Vendor	Intel
Family	Ice Lake
Model	Intel(R) Xeon(R) Gold 6330
Node(s)/Card(s)	1
Socket(s)	2
Cores	56
GHz	2.0
Memory	1000GB DDR4
Sustained BW	320GB/s
LLC	84MB
Sustained BW	1.1TB/s
Compiler	DPC++/C++ compiler 2021.3.0
BLAS library	Intel oneMKL 2021.3.0
MPI library	IntelMPI 2021.3.0

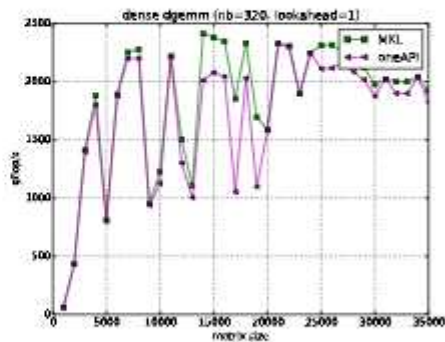
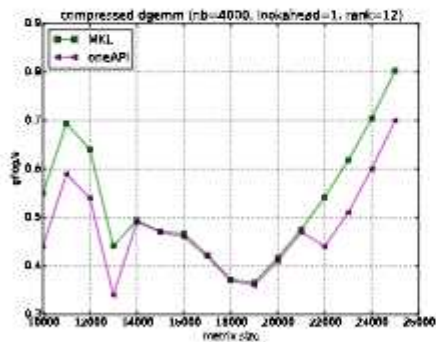
Preliminary Performance Results: TLR GEMM toward MLE Computations in Climate/Weather Prediction (1)

- Tile low-rank GEMM with OpenMP Task



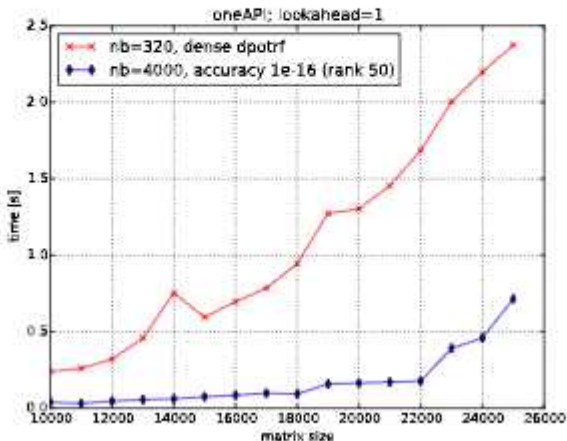
Preliminary Performance Results: TLR GEMM toward MLE Computations in Climate/Weather Prediction (2)

Tile low-rank GEMM with OpenMP Task Vs oneAPI

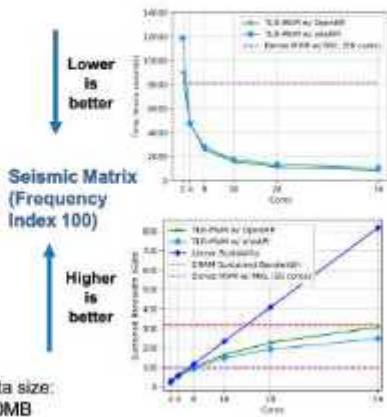
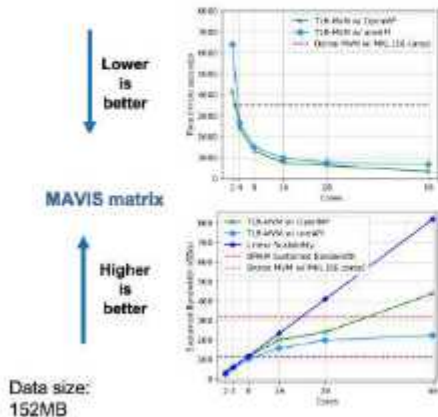


Preliminary Performance Results: TLR Cholesky toward MLE Computations in Climate/Weather Prediction (3)

- Tile low-rank Cholesky with oneAPI



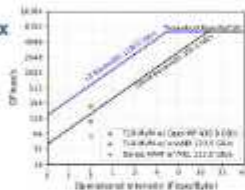
Preliminary Performance Results: TLR-MVM for MAVIS Instrument and Seismic Redatuming



Preliminary Performance Results: TLR-MVM for MAVIS Instrument and Seismic Redatuming

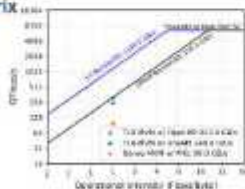
MAVIS Matrix

↑
Higher
is
better



Seismic Matrix
(Frequency Index 100)

↑
Higher
is
better



Operational Intensity (OI) of TLRMVM:

$$OI(fp32) = \frac{FLOPS}{BYTE} = \frac{4Rb}{(n+m+4R+2Rb) \times B} \approx \frac{2}{B} \approx \frac{1}{2}$$

$$OI(complex) = \frac{FLOPS}{BYTE} = \frac{16Rb}{(n+m+4R+2Rb) \times B} \approx \frac{8}{B} \approx 1$$

m, n : rows and columns of input Matrix A

R : Total rank summation of Matrix A at a given threshold

B : Bytes per element : 4 for MAVIS Matrix (single)
8 for Seismic Matrix (single complex)

b : The tile size of small block matrix (256)

Conclusion and Future Work

- Describe a synergistic approach to leverage performance of HPC applications on heterogeneous hardware architectures
- Enhance user-productivity with *oneAPI* programming model
- Study the interoperability between OpenMP/*oneAPI*
- Plan to port on Intel FPGAs and GPUs

Thank You! Questions?

